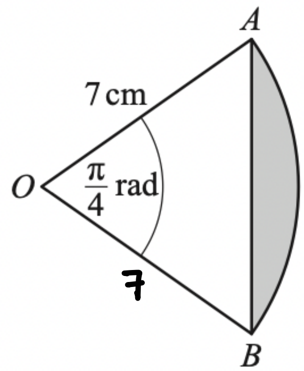
Chapter 8 Circular Measure

1.



The diagram shows the sector *AOB* of a circle with centre *O* and radius 7 cm.
Angle *AOB* = $\frac{\pi}{4}$ radians. Find the perimeter of the shaded region.

[3]

arc length $= r\theta$

$= 7 \times \frac{\pi}{4} = \frac{7\pi}{4}$

$AB^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \times \cos\frac{\pi}{4}$

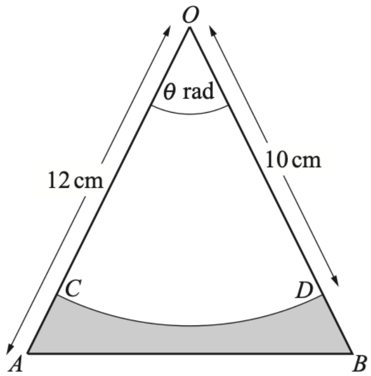
$= 49 + 49 - 49\sqrt{2}$

$AB = 5.358$

Perimeter $= \frac{7\pi}{4} + 5.358$

$= 10.9$ cm

3.



The diagram shows an isosceles triangle *OAB* such that *OA* = *OB* = 12cm and angle *AOB* = θ radians. Points *C* and *D* lie on *OA* and *OB* respectively such that *CD* is an arc of the circle, centre *O*, radius 10 cm. The area of the sector *OCD* = 35cm^2.

a. Show that θ = 0.7 [1]

$\frac{1}{2} \times 100 \times \theta = 35$

$\theta = \frac{35}{50}$

$\theta = 0.7$ (shown)

b. Find the perimeter of the shaded region. [4]

Arc length $= r\theta$

$= 7$ cm

$AB^2 = 12^2 + 12^2 - 2 \times 12 \times 12 \times \cos 0.7$

$AB = 8.23$ cm

$P = 8.23 + 7 + 2 + 2$

$= 19.23$ cm

c. Find the area of the shaded region [3]

Area of $\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 12 \times 12 \times \sin 0.7$

$= 46.4 cm^2$

Area of sector $= \frac{1}{2} r^2 \theta$

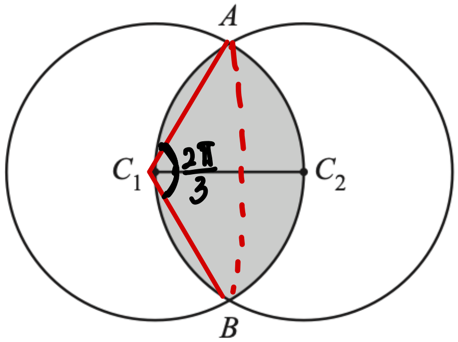
$= \frac{1}{2} \times 100 \times 0.7$

$= 35$ cm^2

shaded $= 46.4 - 35$

$= 11.4 cm^2$

4.



The circles with centres C_1 and C_2 have equal radii of length r cm. The line C_1C_2 is a radius of both circles. The two circles intersect at A and B.

a. Given that the perimeter of the shaded region is 4π cm, find the value of r.

[4]

$\angle AC_1B = \frac{2\pi}{3}$

$P = \frac{2\pi}{3} \times r_1 + \frac{2\pi}{3} r$

$4\pi = \frac{4\pi}{3} r$

$r = 3$ cm

b. Find the exact area of the shaded region.

[4]

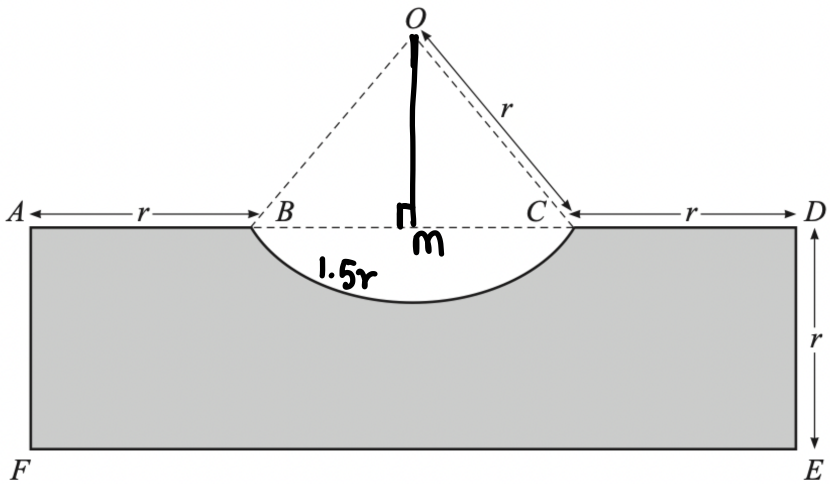
$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 9 \times \frac{2\pi}{3} = 3\pi$ cm^2

$\frac{1}{2} ab \sin c = \frac{1}{2} \times 9 \times \sin \frac{2\pi}{3} = \frac{9}{2} \times \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}$

shaded $= 2 \times \left(3\pi - \frac{9\sqrt{3}}{4}\right)$

$= 6\pi - \frac{9\sqrt{3}}{2}$ cm^2

5. In this question all lengths are in centimetres and all angles are in radians.



The diagram shows the rectangle *ADEF*, where *AF* = *DE* = *r* . The points *B* and *C* lie on *AD* such that *AB* = *CD* = *r*. The curve *BC* is an arc of the circle, centre *O*, radius *r* and has a length of 1.5*r*.

(a) Show that the perimeter of the shaded region is (7.5 + 2 sin 0.75) *r* .

[5]

arc length $= r\theta$

$1.5r = r\theta$

$\theta = 1.5$

$\frac{BM}{r} = \sin 0.75$

$BM = r\sin 0.75$

$BC = 2r\sin 0.75$

$FE = 2r + 2r\sin 0.75$

$P = 2r + 2r\sin 0.75 + 1.5r + 4r$

$= 7.5r + 2r\sin 0.75$

$= (7.5 + 2\sin 0.75)r$ (shown)

(b) Find the area of the shaded region, giving your answer in the form kr^2, where *k* is a constant correct to 2 decimal places.

[4]

Area of $\square = l \times b$

$= r \times 2r + 2r \sin 0.75$

$= 2r^2 + 2r^2 \sin 0.75 = 2r^2 + 1.363r^2$

$= 3.363r^2$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}r^2 \, 1.5$

$= 0.75r^2$

Area of $\triangle = \frac{1}{2}ab\sin c$

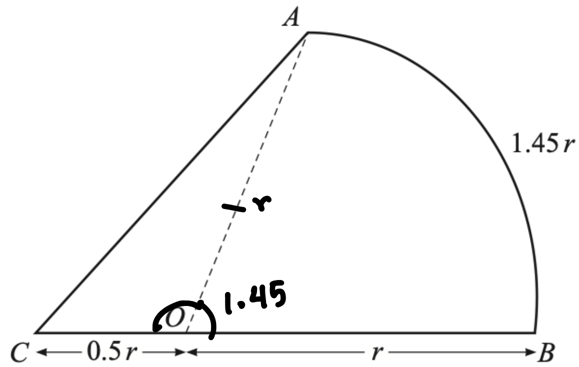
$= \frac{1}{2}r^2 \sin 1.5$

$= 0.499r^2$

shaded area $= 3.363r^2 - (0.75r^2 - 0.499r^2)$

$= 3.11r^2$

6. In this question all lengths are in centimetres.



The diagram shows the figure ABC. The arc AB is part of a circle, centre O, radius r, and is of length $1.45r$. The point O lies on the straight line CB such that $CO = 0.5r$.

a. Find, in radians, the angle AOB.

[1]

Arc $= r\theta$

$1.45r = r\theta$

$\theta = 1.45$

b. Find the area of ABC, giving your answer in the form kr^2, where k is a constant.

[3]

$\angle AOC = \pi - 1.45$

$= 1.692$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times r^2 \times 1.45$

$= 0.725r^2$

Area of $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times r \times 0.5r \times \sin 1.692$

$= 0.248r^2$

total area $= 0.973\,r^2$

c. Given that the perimeter of *ABC* is 12 cm, find the value of *r*.

[4]

$AC^2 = (0.5r)^2 + r^2 - 2 \times 0.5r \times r \times \cos 1.692$

$AC^2 = 0.25r^2 + r^2 + 0.121r^2$

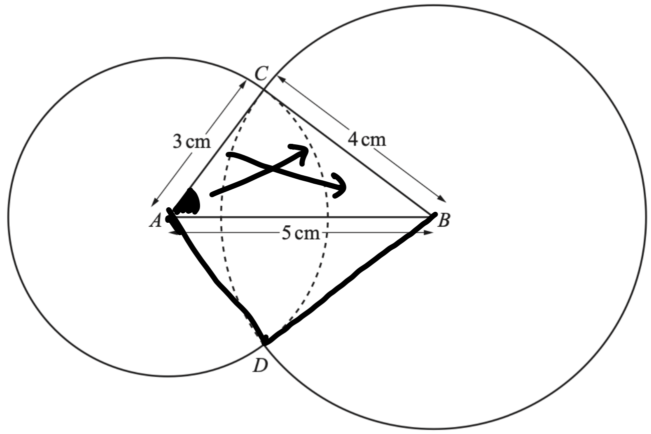
$AC = 1.17r$

$P = 1.17r + 0.5r + r + 1.45r$

$12 = 4.12r$

$r = 2.91$

7.



The diagram shows a shape consisting of two circles of radius 3cm and 4cm with centres *A* and *B* which are 5 cm apart. The circles intersect at *C* and *D* as shown. The lines *AC* and *BC* are tangents to the circles, centres *B* and *A* respectively. Find

a. the angle *CAB* in radians, [2]

$4^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos A$

$16 = 9 + 25 - 30\cos A$

$-18 = -30\cos A$

$\cos A = \frac{3}{5}$

$A = \cos^{-1}\left(\frac{3}{5}\right)$

$= 0.927$

b. the perimeter of the whole shape [4]

$\angle ACB = 2\pi - 0.927 \times 2$

$= 4.429$

Arc length $= r\theta$

$= 3 \times 4.429$

$= 13.29$ cm

$\frac{\sin 0.927}{4} = \frac{\sin \theta}{3}$

$\theta = 0.6433$

$\angle CBD = 2\pi - 0.6433 \times 2$

$= 4.997$

Arc $= r\theta$

$= 4 \times 4.997 = 19.99$

$P = 13.29 + 19.99$

$= 33.3$ cm

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c. the area of the whole shape.

[4]

$$\text{sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 3^2 \times 4.429$$
$$= 19.9305$$

$$\text{sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 4^2 \times 4.997$$
$$= 39.976$$

$$2\Delta = \frac{1}{2}ab\sin C \times 2$$
$$= 4 \times 5 \times \sin 0.6433$$
$$= 11.997$$

$$\text{total area} = 19.93 + 39.98 + 12$$
$$= 71.9\ \text{cm}^2$$